

NAG C Library Function Document

nag_dtrrfs (f07thc)

1 Purpose

nag_dtrrfs (f07thc) returns error bounds for the solution of a real triangular system of linear equations with multiple right-hand sides, $AX = B$ or $A^T X = B$.

2 Specification

```
void nag_dtrrfs (Nag_OrderType order, Nag_UploType uplo, Nag_TransType trans,
                 Nag_DiagType diag, Integer n, Integer nrhs, const double a[], Integer pda,
                 const double b[], Integer pdb, const double x[], Integer pdx, double ferr[],
                 double berr[], NagError *fail)
```

3 Description

nag_dtrrfs (f07thc) returns the backward errors and estimated bounds on the forward errors for the solution of a real triangular system of linear equations with multiple right-hand sides $AX = B$ or $A^T X = B$. The function handles each right-hand side vector (stored as a column of the matrix B) independently, so we describe the function of nag_dtrrfs (f07thc) in terms of a single right-hand side b and solution x .

Given a computed solution x , the function computes the *component-wise backward error* β . This is the size of the smallest relative perturbation in each element of A and b such that x is the exact solution of a perturbed system

$$(A + \delta A)x = b + \delta b$$

$$|\delta a_{ij}| \leq \beta |a_{ij}| \quad \text{and} \quad |\delta b_i| \leq \beta |b_i|.$$

Then the function estimates a bound for the *component-wise forward error* in the computed solution, defined by:

$$\max_i |x_i - \hat{x}_i| / \max_i |x_i|$$

where $\hat{x}$ is the true solution.

For details of the method, see the f07 Chapter Introduction.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

- 1: **order** – Nag_OrderType *Input*
On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.
Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.
- 2: **uplo** – Nag_UploType *Input*
On entry: indicates whether A is upper or lower triangular as follows:

if **uplo** = **Nag_Upper**, then A is upper triangular;

if **uplo** = **Nag_Lower**, then A is lower triangular.

Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.

3: **trans** – Nag_TransType *Input*

On entry: indicates the form of the equations as follows:

if **trans** = **Nag_NoTrans**, then the equations are of the form $AX = B$;

if **trans** = **Nag_Trans** or **Nag_ConjTrans**, then the equations are of the form $A^T X = B$.

Constraint: **trans** = **Nag_NoTrans**, **Nag_Trans** or **Nag_ConjTrans**.

4: **diag** – Nag_DiagType *Input*

On entry: indicates whether A is a non-unit or unit triangular matrix as follows:

if **diag** = **Nag_NonUnitDiag**, A is a non-unit triangular matrix;

if **diag** = **Nag_UnitDiag**, A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = **Nag_NonUnitDiag** or **Nag_UnitDiag**.

5: **n** – Integer *Input*

On entry: n , the order of the matrix A .

Constraint: $n \geq 0$.

6: **nrhs** – Integer *Input*

On entry: r , the number of right-hand sides.

Constraint: **nrhs** ≥ 0 .

7: **a**[*dim*] – const double *Input*

Note: the dimension, *dim*, of the array **a** must be at least $\max(1, \mathbf{pda} \times \mathbf{n})$.

On entry: the n by n triangular matrix A . If **uplo** = **Nag_Upper**, A is upper triangular and the elements of the array below the diagonal are not referenced; if **uplo** = **Nag_Lower**, A is lower triangular and the elements of the array above the diagonal are not referenced. If **diag** = **Nag_UnitDiag**, the diagonal elements of A are not referenced, but are assumed to be 1.

8: **pda** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix A in the array **a**.

Constraint: **pda** $\geq \max(1, \mathbf{n})$.

9: **b**[*dim*] – const double *Input*

Note: the dimension, *dim*, of the array **b** must be at least $\max(1, \mathbf{pdb} \times \mathbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \mathbf{pda} \times \mathbf{n})$ when **order** = **Nag_RowMajor**.

If **order** = **Nag_ColMajor**, the (i, j) th element of the matrix B is stored in **b**[($j - 1$) $\times$ **pdb** + $i - 1$] and if **order** = **Nag_RowMajor**, the (i, j) th element of the matrix B is stored in **b**[($i - 1$) $\times$ **pda** + $j - 1$].

On entry: the n by r right-hand side matrix B .

10: **pdb** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = **Nag_ColMajor**, **pdb** $\geq \max(1, \mathbf{n})$;
 if **order** = **Nag_RowMajor**, **pdb** $\geq \max(1, \mathbf{nrhs})$.

11: **x**[*dim*] – const double

Input

Note: the dimension, *dim*, of the array **x** must be at least $\max(1, \mathbf{pdx} \times \mathbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \mathbf{pdx} \times \mathbf{n})$ when **order** = **Nag_RowMajor**.

If **order** = **Nag_ColMajor**, the (*i*, *j*)th element of the matrix *X* is stored in **x**[(*j* – 1) $\times$ **pdx** + *i* – 1] and if **order** = **Nag_RowMajor**, the (*i*, *j*)th element of the matrix *X* is stored in **x**[(*i* – 1) $\times$ **pdx** + *j* – 1].

On entry: the *n* by *r* solution matrix *X*, as returned by nag_dtrtrs (f07tec).

12: **pdx** – Integer

Input

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **x**.

Constraints:

if **order** = **Nag_ColMajor**, **pdx** $\geq \max(1, \mathbf{n})$;
 if **order** = **Nag_RowMajor**, **pdx** $\geq \max(1, \mathbf{nrhs})$.

13: **ferr**[*dim*] – double

Output

Note: the dimension, *dim*, of the array **ferr** must be at least $\max(1, \mathbf{nrhs})$.

On exit: **ferr**[*j* – 1] contains an estimated error bound for the *j*th solution vector, that is, the *j*th column of *X*, for *j* = 1, 2, ..., *r*.

14: **berr**[*dim*] – double

Output

Note: the dimension, *dim*, of the array **berr** must be at least $\max(1, \mathbf{nrhs})$.

On exit: **berr**[*j* – 1] contains the component-wise backward error bound β for the *j*th solution vector, that is, the *j*th column of *X*, for *j* = 1, 2, ..., *r*.

15: **fail** – NagError *

Output

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = *<value>*.

Constraint: **n** ≥ 0 .

On entry, **nrhs** = *<value>*.

Constraint: **nrhs** ≥ 0 .

On entry, **pda** = *<value>*.

Constraint: **pda** > 0 .

On entry, **pdb** = *<value>*.

Constraint: **pdb** > 0 .

On entry, **pdx** = *<value>*.

Constraint: **pdx** > 0 .

NE_INT_2

On entry, **pda** = *<value>*, **n** = *<value>*.

Constraint: **pda** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = $\langle value \rangle$, **n** = $\langle value \rangle$.

Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = $\langle value \rangle$, **nrhs** = $\langle value \rangle$.

Constraint: **pdb** $\geq \max(1, \mathbf{nrhs})$.

On entry, **pdx** = $\langle value \rangle$, **n** = $\langle value \rangle$.

Constraint: **pdx** $\geq \max(1, \mathbf{n})$.

On entry, **pdx** = $\langle value \rangle$, **nrhs** = $\langle value \rangle$.

Constraint: **pdx** $\geq \max(1, \mathbf{nrhs})$.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The bounds returned in **ferr** are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

8 Further Comments

A call to nag_dtrrfs (f07thc) involves, for each right-hand side, solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately n^2 floating-point operations.

The complex analogue of this function is nag_ztrrfs (f07tvc).

9 Example

To solve the system of equations $AX = B$ and to compute forward and backward error bounds, where

$$A = \begin{pmatrix} 4.30 & 0.00 & 0.00 & 0.00 \\ -3.96 & -4.87 & 0.00 & 0.00 \\ 0.40 & 0.31 & -8.02 & 0.00 \\ -0.27 & 0.07 & -5.95 & 0.12 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -12.90 & -21.50 \\ 16.75 & 14.93 \\ -17.55 & 6.33 \\ -11.04 & 8.09 \end{pmatrix}.$$

9.1 Program Text

```
/* nag_dtrrfs (f07thc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
```

```

/* Scalars */
Integer i, j, n, nrhs, berr_len, ferr_len;
Integer pda, pdb, pdx;
Integer exit_status=0;
Nag_UploType uplo_enum;

NagError fail;
Nag_OrderType order;
/* Arrays */
char uplo[2];
double *a=0, *b=0, *berr=0, *ferr=0, *x=0;

#ifdef NAG_COLUMN_MAJOR
#define A(I,J) a[(J-1)*pda + I - 1]
#define B(I,J) b[(J-1)*pdb + I - 1]
#define X(I,J) x[(J-1)*pdx + I - 1]
    order = Nag_ColMajor;
#else
#define A(I,J) a[(I-1)*pda + J - 1]
#define B(I,J) b[(I-1)*pdb + J - 1]
#define X(I,J) x[(I-1)*pdx + J - 1]
    order = Nag_RowMajor;
#endif

INIT_FAIL(fail);
Vprintf("f07thc Example Program Results\n\n");

/* Skip heading in data file */
Vscanf("%*[^\\n] ");
Vscanf("%ld%ld%*[^\\n] ", &n, &nrhs);
berr_len = nrhs;
ferr_len = nrhs;
#ifdef NAG_COLUMN_MAJOR
    pda = n;
    pdb = n;
    pdx = n;
#else
    pda = n;
    pdb = nrhs;
    pdx = nrhs;
#endif

/* Allocate memory */
if ( !(a = NAG_ALLOC(n * n, double)) ||
      !(b = NAG_ALLOC(n * nrhs, double)) ||
      !(berr = NAG_ALLOC(berr_len, double)) ||
      !(ferr = NAG_ALLOC(ferr_len, double)) ||
      !(x = NAG_ALLOC(n * nrhs, double)) )
{
    Vprintf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read A and B from data file, and copy B to X */
Vscanf(" ' %ls '%*[^\\n] ", uplo);
if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
else if (*(unsigned char *)uplo == 'U')
    uplo_enum = Nag_Upper;
else
{
    Vprintf("Unrecognised character for Nag_UploType type\n");
    exit_status = -1;
    goto END;
}
if (uplo_enum == Nag_Upper)
{
    for (i = 1; i <= n; ++i)
    {

```

```

        for (j = i; j <= n; ++j)
            Vscanf("%lf", &A(i,j));
    }
    Vscanf("%*[^\\n] ");
}
else
{
    for (i = 1; i <= n; ++i)
    {
        for (j = 1; j <= i; ++j)
            Vscanf("%lf", &A(i,j));
    }
    Vscanf("%*[^\\n] ");
}
for (i = 1; i <= n; ++i)
{
    for (j = 1; j <= nrhs; ++j)
        Vscanf("%lf", &B(i,j));
}
Vscanf("%*[^\\n] ");
for (i = 1; i <= n; ++i)
{
    for (j = 1; j <= nrhs; ++j)
        X(i,j) = B(i,j);
}

/* Compute solution in the array X */
f07tec(order, uplo_enum, Nag_NoTrans, Nag_NonUnitDiag, n,
        nrhs, a, pda, x, pdx, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f07tec.%s\\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Compute backward errors and estimated bounds on the */
/* forward errors */

f07thc(order, uplo_enum, Nag_NoTrans, Nag_NonUnitDiag, n,
        nrhs, a, pda, b, pdb, x, pdx, ferr, berr, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f07thc.%s\\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print solution */

Vprintf("\\n");
x04cac(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, nrhs,
        x, pdx, "Solution(s)", 0, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from x04cac.%s\\n", fail.message);
    exit_status = 1;
    goto END;
}
Vprintf("\\nBackward errors (machine-dependent)\\n");

for (j = 1; j <= nrhs; ++j)
    Vprintf("%11.1e%s", berr[j-1], j%7==0 ? "\\n": " ");
Vprintf("\\nEstimated forward error bounds "
        "(machine-dependent)\\n");
for (j = 1; j <= nrhs; ++j)
    Vprintf("%11.1e%s", ferr[j-1], j%7==0 ? "\\n": " ");
Vprintf("\\n");
END:
if (a) NAG_FREE(a);
if (b) NAG_FREE(b);

```

```

    if (berr) NAG_FREE(berr);
    if (ferr) NAG_FREE(ferr);
    if (x) NAG_FREE(x);

    return exit_status;
}

```

9.2 Program Data

f07thc Example Program Data

```

    4  2                                :Values of N and NRHS
    'L'                                :Value of UPLO
    4.30
    -3.96 -4.87
    0.40  0.31 -8.02
    -0.27  0.07 -5.95  0.12          :End of matrix A
    -12.90 -21.50
    16.75  14.93
    -17.55  6.33
    -11.04  8.09                      :End of matrix B

```

9.3 Program Results

f07thc Example Program Results

Solution(s)

	1	2
1	-3.0000	-5.0000
2	-1.0000	1.0000
3	2.0000	-1.0000
4	1.0000	6.0000

Backward errors (machine-dependent)

6.9e-17 0.0e+00

Estimated forward error bounds (machine-dependent)

8.3e-14 2.6e-14
